On the Industrialization and Alienation of Math Education

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Mother nature is a creature of habit. She follows her routine with precision and abhors deviation. The job of a physicist is to uncover, quantify, and explain mother nature's habits, commonly referred to as "laws." Physics is an encompassing field, with all roads of science leading to it. Its interests range from the fundamental quarks and leptons of quantum field theories to the entire structure and evolution of the universe itself. Thus, there is a division of labor in the field of physics. There are theorists and experimentalists who specialize in a specific subfield of physics (cosmology, nuclear, elementary particle, etc.). According to the Nobel-prize-winning physicist Richard Feynman, physicists "look for a new law by the following process. First, we guess it… Then we compute the consequences of the guess… we see what it would imply. And then we compare those computation results to nature." The job of a theorist is that of a professional guesser. They use their main tool, mathematics, to guess the new laws and theories. They then "compute the consequences" (Feynman) and thus produce a falsifiable prediction that they hand over to the experimentalist, a physicist specializing in proving theorists wrong. Since humans are incapable of understanding the entire universe directly, we create models in the likeness of the universe that we can understand. Much like how a sculptor must look away from their art and at the subject to make sure their sculpture is accurate, so too must an experimentalist look at the universe to ensure the theorist's models are accurate. Therefore, the falsifiability of a model is important. If a model can be proven wrong, we call it "physics". If it can't, we call it "string theory."

In what is quite possibly the greatest plot twist in the story of humanity, mathematics, the method our ancestors created for keeping track of food and trading animals, is unparalleled in its ability to create models of galaxies, atoms, and other evolving systems. This discovery was so profound to humans throughout history that it was taken as divine revelation. Before LSD, looking at a times table was the closest way to reach God. The Platonic Solids have received considerable esoteric attention. A Platonic solid is a three-dimensional solid whose faces are made up of the same polygon that meet at identical vertices. In his 1596 book "Mysterium Cosmographicum", Johannes Kepler stated that he believed God designed the planet's orbits in accordance with the ratios found when encasing Platonic solids inside one another.

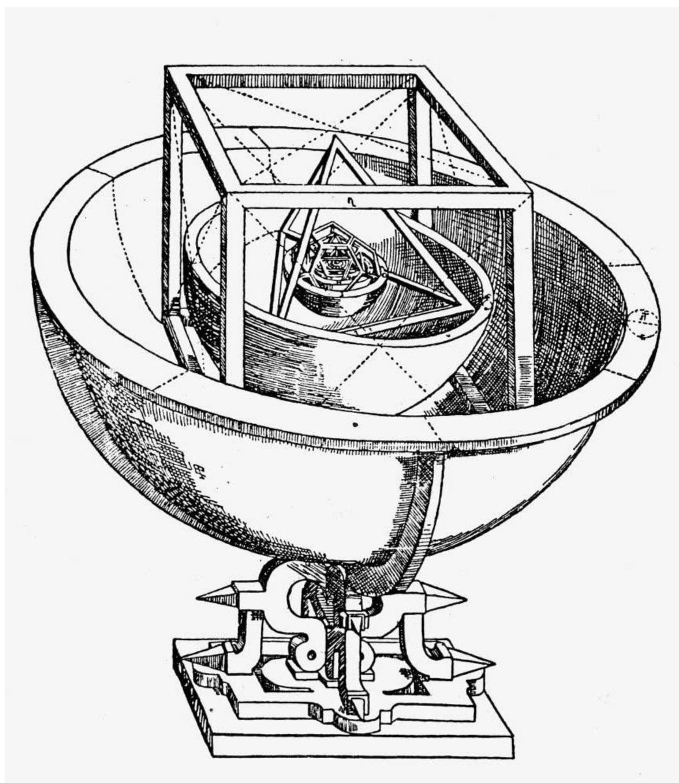


Fig.1 Kepler's model of the Solar System from his book "Mysterium Cosmographicum"

Thus, geometry was seen as more than a mere method of describing nature; it was divine. Kepler's theory was tested by the observations of famed astronomer Tycho Brahe in the early

17th century. Like a sound experimentalist, his work proved Kepler's theory wrong. Kepler found that the planets' distances from the sun vary and thus cannot be described by platonic ratios.

Some forms of mathematics are criticized for being "too abstract" and for having no application. However, even mathematics that initially comes across as mere intellectual play tends to find itself accurately describing reality sometime in the future. The ancient Greeks had a great interest in the study of conics, which is the study of the cross-sections of cones. Depending on the angle of the cross-section, it produces different shapes.

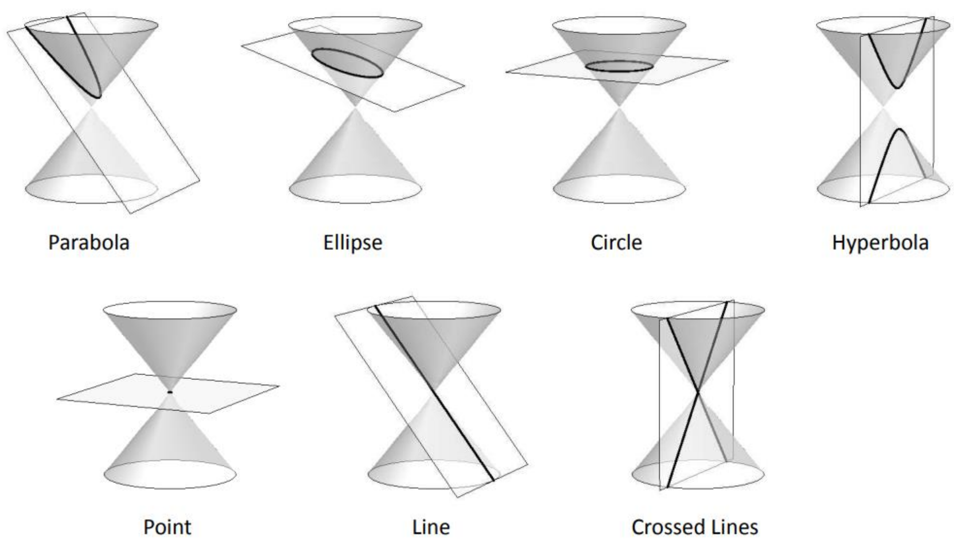


Fig 2. Conic Sections (Gregory Hartman et al.)

For the longest time, this study was seen as entirely inapplicable, prompting students in ancient Athens to ask their teachers "When am I ever going to use this in real life?", except they probably said it in Greek. It wasn't until Kepler's idea of platonic orbits was disproven that he found that the actual orbits of the planets could be described mathematically using ellipses, a type of conic section. Finally, the mathematics of conics found a real-world application (and it only took 2000 years)!

It would seem that math is so good at describing reality that it does it by accident. However, math need not have a direct real-world application to be worth the time we make our children learn it. Math is the purest form of logical reasoning and trains our abstract thinking ability. In 2020, researchers from Australia found in their study that “in general the greater the mathematics training of the participant, the more tasks were completed correctly, and that performance on some tasks was also associated with performance on others not traditionally associated.” (Cresswell and Speelman). Math, and thus critical thinking skills, must be impressed upon children as young as possible. Children who are not taught to reason grow up to be teens who cannot reason, who then grow up to be members of Congress.

Despite this horrifying prospect, students leave school without an appreciation for math and mathematical thinking. It is no exaggeration that our society runs on mathematics (literally, in the case of architecture), yet many adults struggle with even simple algebra. In his cleverly titled book “Alex’s Adventure’s in Numberland”, Alex Bellos recalls an anecdote from when he was a cub reporter at The Argus, a newspaper company in Brighton, England. In June 1992, Prime Minister John Major introduced a government hotline for citizens to inquire about traffic cones. The policy was mocked due to how seriously it was being spun by Major, and how boring it was after the whirlwind of a political scene Britain had under Margaret Thatcher. Alex Bellos writes “With its tongue firmly in its cheek, the *Argus* challenged its readers to guess the number of cones that lines the many miles of the A23 (M)” (8). Imagine the surprise on the faces of the editorial board when a caller gave the right answer a few hours after the competition was made public. The answer is quite easy: divide the road’s length by the average length between the cones. These full-grown adults, most of whom graduated from college, were unable to apply elementary math in the real world. The story is one of harmless embarrassment, but what about those mathematically illiterate adults who hold positions of great importance?

But why is it so common that we see children and adults who feel alienated from mathematics? It is partly due to the fact that mathematics is an inherently abstract mode of thought that sometimes goes against our ape brains. Let's start as simple as possible with the concept of numbers and counting. In his book "The Joy of x", Steven Strogatz talks about a 1997 episode of Sesame Street called "123 Count With Me." In it, a Muppet named Humphrey receives a call from one of the rooms in his hotel. On the other side, six penguins say that they are hungry, and they all want a fish each. Relaying the message to an employee, Humphrey says "go tell the kitchen 'fish, fish, fish, fish, fish, fish'" (16:02). With such a mouthful of a request, the employee naturally becomes confused as to how many fish the cooks need to make. This prompts Ernie to make a suggestion to Humphrey. Instead of saying "fish" for every penguin, Ernie tells Humphrey that he can say "six fish". This episode shows the time-saving ability of numbers when representing physical quantities, especially when the quantity reaches double or even triple digits. But Strogatz writes that numbers are an abstraction by their very nature. "Six is more ethereal than six fish… It applies to six of anything: six plates, six penguins, sit utterances of the word 'fish.' It's the ineffable thing they all have in common" (Strogatz 4). The abstraction of 3 physical apples in front of you into a symbol "3", even when there exist no real apples, is not a very intuitive jump (and it gets even harder for zero and negative numbers). This abstraction is then taken even further by representing these ethereal symbols has having a definite position on a one-dimensional line with constant displacement where each value x_n is a function of the previous values ($x_n = x_{n-1} + 1$), which is an explanation of the standard number line that is as unintuitive as the number line itself is to very young children. Alex Bellos writes in the zeroth chapter of his aforementioned book about a linguist named Pierre Pica who spent months in the Brazilian Amazon with an isolated indigenous tribe named the Munduruku. Pica found that this tribe does not have words for numbers greater than five, not because they didn't watch Sesame Street, but rather because the concept of counting is absurd to a Munduruku. They live in strong communal tribes where there is no need to keep track of property or its trade. Therefore, the need for counting never arose. With such a group that has been isolated from our formal math education, Pica wanted to run an experiment. He gave the Munduruku a linear line and ten circles, each containing dots. The first circle contained one dot, the second two dots, and so on up to ten dots. The tribe members were asked to place these circles on the line in ascending order according to where they thought they belonged relative to each other. If you were to run this experiment on most adults with a math education, they would place the dots on the line evenly spaced. But when Pica ran this experiment on the Munduruku, they consistently placed the dots on the line logarithmically, with spaces between the circles decreasing as the number of dots in the circles increased.

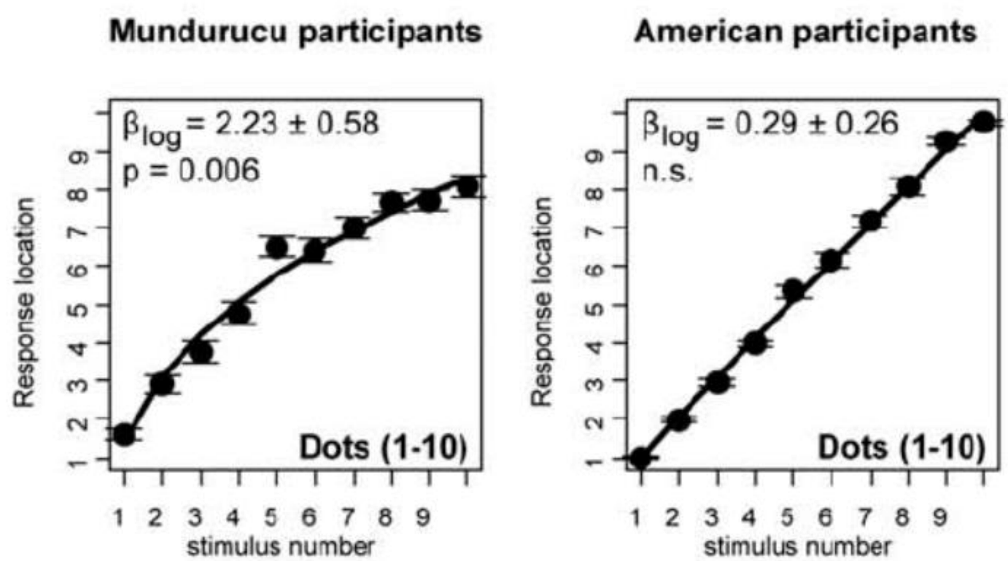


Fig. 3 Pica's results from his experiment with the Munduruku compared to Americans (Dehaene, Stanislas et al.)

This wasn't just a product of tribal life, however. In 2004, researchers from Carnegie Mellon University found that children also place dots on the line logarithmically until they receive a formal math education in the first and second grades (Siegler and Booth). According to Pica, there is a simple evolutionary explanation. We've evolved to see the world in ratios. When the Munduruku see a circle with five dots, they see five times as many dots as the circle with one dot. But when they see a circle with ten dots, they only see a circle with two times as many dots as the circle with five dots. This is despite the fact that the difference between both pairs of circles is five dots. To our ancestors, there was a greater difference in survivability between fighting one tiger and fighting two tigers, than there was between fighting nine tigers and fighting ten tigers. A linear line of ordered abstract symbols, which serves as the basis of math, causes confusion to young students. As they go through their math careers, this problem of nonintuitive abstraction only exacerbates when those symbols begin to represent things that don't exist in this universe (like zero or negative 5 apples).

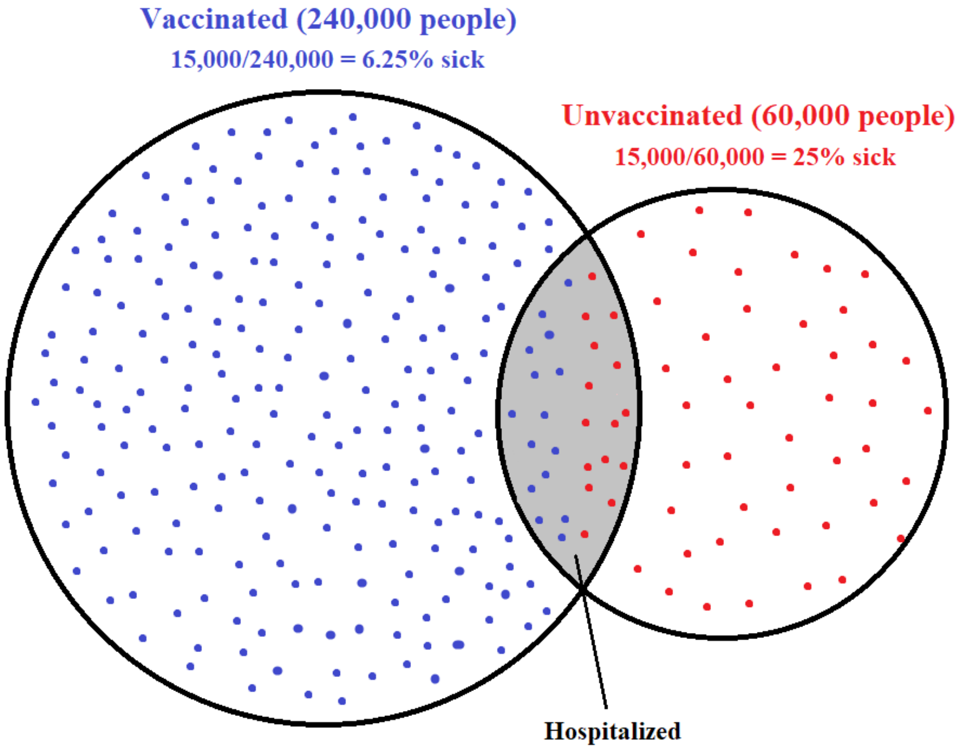
So how does formal math education deal with the problem of abstraction and lack of intuition? Simple, they make the concepts more abstract and give up on intuition. In an attempt to replicate our methods of commodity production, a large portion of math education has industrialized the learning process. Like the factories in the Industrial Revolution, math education has stripped away the difficult craftsmanship of mathematical problem solving and replaced it with a faster and easier method of mechanical memorization and regurgitation. Students are given mathematical theorems to remember with little reasoning behind them. All that the student is told is that it solves a problem without the student realizing *why* it is a problem. The mechanics of how to use an undiscernible theorem to solve a seemingly purposeless problem is the antithesis of mathematics, both pure and applied, and only serves to alienate students from the entire field.

In 1980, the philosopher John Searle sought to disprove the idea that an artificial intelligence can truly think (418). He asks us to imagine a man who can only speak English in a closed room with a large rule book in English and boxes of Chinese characters. A Chinese-speaker outside the room slips in a piece of paper with a question written in Chinese. The man in the room cannot understand a single word of the question. But by consulting the rule book, he is told, in English, what Chinese characters to select (example: 'if you receive "你好吗", respond with "我很好"'). The man follows the rule book and selects the correct Chinese characters and pushes them out the room. The Chinese-speaker on the other side receives the response and falsely believes they are speaking with someone able to understand Chinese. While Searle intended this argument to be used against "Strong AI", it works equally well against formal math education. Math, like Chinese, is a language. And like the man in the room, the average math student does not actually understand the language that they are using, merely the mechanical instructions. The man in the room will forget these instructions and be incapable of holding a real conversation in Chinese. Likewise, the student will forget the enigmatic theorems in their textbooks and be incapable of mathematical reasoning and real-world application. Thus, the ideal of a mathematically literate population is not reached.

YouthTruth, a national nonprofit for surveying students, asked almost 90,000 about their experiences over the 2022-2023 school year. They found that "Many high school students perceive 'school math' or 'education math' as lacking inherent value and as disconnected from their practical needs. Students express a strong desire to acquire 'real' math skills that will empower them to pursue the future they envision for themselves" (YouthTruth 2). The mechanical teaching of math has failed to teach students the importance of a math education at the level of algebra and above. By no fault of the students, they seek to learn math that will help them in adult life, without realizing the algebra and calculus behind filing taxes, making investments, running a business, budgeting, cooking, or raising a child. And that excludes the innumerable STEM and medical applications. Students are also not taught the importance of problem-solving skills that pure mathematics offers in the real world because most of the time it is not even used in the classroom. This poor teaching method has sullied the reputation of "school math" in the eyes of those it would help the most.

As we are living in what is commonly referred to as the "Golden Age of Misinformation", I would like to emphasize the importance of critical thinking that a proper math education offers, especially in the form of statistics. A lie that uses statistics offers it a false sense of scholarly authenticity that many people are too intimidated to challenge. In a world of mathematically illiterate adults, demagogues and charlatans can easily manipulate thousands or even millions, all with a simple Microsoft Excel account. This may seem like a math nerd making a mountain out of a molehill, but I hope to convince you otherwise. Allow me to give a hypothetical that closely parallels what happened a few years ago. Imagine that a town of 300,000 people is experiencing an unspecified pandemic for which they have a vaccine. A survey was taken at every hospital in town and found that out of the 30,000 patients with the disease, 50% of them were vaccinated, and the other 50% were not. Those who lack a proper math education, or those with questionable motives, may falsely concluded from this survey that the vaccine lacks any efficacy and that you have just as much chance of catching the disease regardless of your immunization status. What is missing from these statistics necessary to make such a conclusion are the denominators. Imagine that 80% of people in the town were vaccinated (240,000 people), leaving the remaining 60,000 inhabitants unvaccinated. This would mean that only 15,000 (50% of the patients) out of 240,000 vaccinated citizens got the disease, while

15,000 out of 60,000 unvaccinated people were infected. This means that in this hypothetical, the unvaccinated were four times more likely to be hospitalized.



With such mathematical illiteracy, fearmongers in real life can convinced large portions of the population to avoid immunization or other life-saving procedures because in high school they believed “math was not helpful.”

There is another dangerous myth floating around, almost as dangerous as the one that claims that vaccines are not helpful. The myth that is the progenitor of many of the problems I’ve worn my fingers out typing about in this essay. While mathematics being inherently abstract and sometimes unintuitive may be the genesis of these problems, they are kept alive today by the myth that some people are just not cut out to be a “math person.” The dark side of this myth is that it is often used as an excuse by educators to give up on certain students, and for students to give up on themselves. And I hope by now I’ve convinced the reader of the consequences for a general public who has given up on learning math. While there are those who have an easier start, most math is completely accessible to the average person, regardless of its abstract nature. In a paper published by Nancy Dejarnette in the journal Education, “Research has shown that early exposure to STEM initiatives and activities positively impacts elementary students' perceptions and dispositions” (1).

As previously stated, a math education is imperative to a functioning society that is capable of resisting manipulation. The UN General Assembly declared in 1948 that “Everyone has the right to education… Technical and professional education shall be made generally available” (art. 26). If we recognize the power of being able to read in resisting authoritarianism, we must accept the power of mathematical literacy, because a dictator is just as capable of lying through their numbers as they are through their words.

Reflecting on the troubled waters of math education as a math student working towards a physics career in academia, I plan to adopt a mode of teaching different from what I’ve experienced in the past. I want to replicate the physicists and mathematicians I admire and show my future students that math is not to be feared. It is a liberating tool that can quantify even the most seemingly chaotic system. I want to teach them math as a system of analytical thought, and how that system has evolved to meet the problems of the world and mind. Most importantly, I want to show people that math is beautiful.

Works Cited

Feynmann, Richard. "Seeking New Laws." 9 Nov. 1964, Cornell University, Ithaca. Messenger Lectures.

Kepler, Johannes. *Mysterium Cosmographicum*. Tübingen, 1596.

Hartman, Gregory et al., "9.1: Conic Sections" *LibreTexts Mathematics*, https://math.libretexts.org/Bookshelves/Calculus/Calculus_3e_%28Apex%29/09%3A_Curves_in_the_Plane/9.01%3A_Conic_Sections

Cresswell, Clio., and Craig Speelman. "Does mathematics training lead to better logical thinking and reasoning? A cross-sectional assessment from students to professors." *PLoS One*, vol. 15, no. 7, 2020, e0236153.

Bellos, Alex. *Alex's Adventures in Numberland*. London, Bloomsbury Publishing, 2010.

Strogatz, Steven. *The Joy of x*. New York, HarperCollins, 2012.

"123 Count With Me." Sesame Street, created by Ted May et al., PBS, 1997.

Dehaene, Stanislas et al., "Log or Linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures." *Science*, vol. 320, no. 5880, 2008, pp. 1217 – 1220.

Siegler, Robert S., and Julie L. Booth. "Development of Numerical Estimation in Young Children." *Child Development*, vol. 75, no. 2, 2004, pp. 428 – 444.

Searle, John. "Minds, brains, and programs." *The Behavioral and Brain Sciences*, vol. 3, no. 3, 1980, pp. 417 – 457.

"Making Sense of Learning Math Insights from the Student Experience" *YouthTruth*, The Center for Effective Philanthropy, 2023, youthtruthsurvey.org/wp-content/uploads/2024/01/Making-Sense-of-Learning-Math_2023.pdf.

Sutcher, Leib, et al. *A Coming Crisis in Teaching? Teacher Supply, Demand, and Shortages in the U.S.* Learn Policy Institute, 15 Sept. 2016. https://learningpolicyinstitute.org/product/coming-crisis-teaching.

DeJarnette, Nancy K. "America's Children: Providing early exposure to STEM (Science, Technology, Engineering, & Math) Initiatives." *Education*, vol. 133, no. 1, 2012, pp. 77 – 84.

UN General Assembly. "Universal Declaration of Human Rights." United Nations, 217 (III) A, 1948, Paris, art. 26, http://www.un.org/en/universal-declaration-human-rights/. Accessed 19 April 2024.